The World of Small Particles

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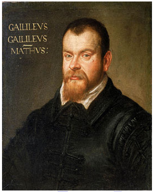
Overview

Pioneers of Classical Mechanics

- Nicolaus Copernicus (1473–1543) proposed the heliocentric model

- Johannes Kepler (1571–1630) known for Kepler's laws of planetary motion

- Galileo Galilei (1564–1642) central figure of the scientific revolution of the 17^{th} century, known for kinematics, analytical dynamics, observational astronomy



Pioneers of Classical Mechanics

- Isaac Newton (1642–1727) known for Newton's laws of motion, theory of gravitation, calculus etc.

- Leonhard Euler (1707–1783) known for Theory of the motions of rigid bodies, analytic geometry, trigonometry, calculus and number theory

Other important physicists: Tycho Brahe, Christiaan Huygens, Joseph-Louis Lagrange, William Rowan Hamilton

Theory of light

- Corpuscular theory of light $\longrightarrow$ Isaac Newton (1642–1727)

- Wave theory of light $\longrightarrow$ Christiaan Huygens (1629–1695)

- Electromagnetic theory $\longrightarrow$ James Clerk Maxwell (1831–1879)

Origins of quantum mechanics

- Classical mechanics $\longrightarrow$ very successful in describing motion of macroscopic objects

- Wave theory of light $\longrightarrow$ successful in explaining interference, diffraction, polarization but fails to explain photoelectric effect, Compton effect

- Investigation of black body radiation by Max Planck (1900) $\longrightarrow$ introduction of Planck constant h $\longrightarrow$ energy is quantized $\mathcal{E} = h\nu$, where ν is the frequency of radiation

Development of Quantum Mechanics

Milestones

- Photoelectric effect $\longrightarrow$ Albert Einstein (1905)
- Theory of specific heats of solids at low temperature $\longrightarrow$ Albert Einstein (1907)
- Solvay Congress (1^{st}) (1911)
- Bohr model of the atom $\longrightarrow$ Niels Bohr (1913) (successful in explaining spectral lines)
- Einstein's papers on radiative transitions (connecting Bohr atom back to blackbody radiation) (1917)
- Compton effect $\longrightarrow$ Arthur Compton (1923)

Modern Quantum Mechanics

Milestones

- Wave-particle duality $\longrightarrow$ Louis de Broglie (1923)
- Equation for the time-evolution of a wave $\longrightarrow$ Erwin Schrödinger (1926)
- Uncertainty equation $\longrightarrow$ Werner Heisenberg (1927)
- Solvay Congress (5^{th}) (1927)

Important Contributions: Max Born, Pascual Jordan, Wolfgang Pauli, Paul Dirac and many others.

Some Thought Experiments

How do small particles behave? How does light behave?

Small particles $\longrightarrow$ particles ... waves.
Light $\longrightarrow$ particles ... waves.
They behave like neither. But small particles behave just like light.

Quantum Mechanics $\longrightarrow$ description of the behavior of matter and light

Human intuition applies to large objects $\longrightarrow$ description of the behavior of small particles is highly **non-intuitive**

Let us learn from "thought" experiments.

An experiment with bullets

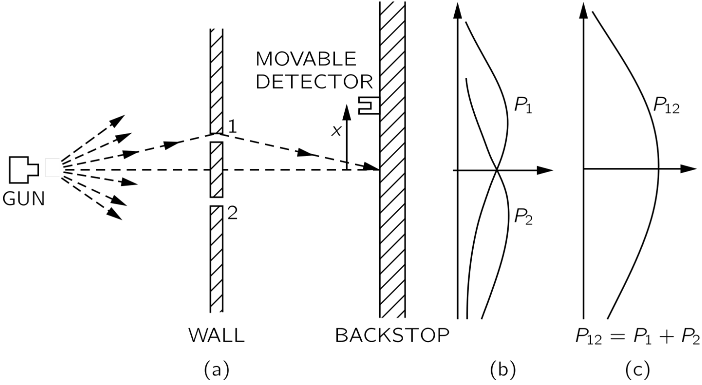


Figure: Interference experiment with bullets

- Holes are big enough to let the bullets through
- What is the probability that a bullet that passes through the holes will arrive at the backstop at a distance x from the center?
- We can only talk about **probabilities**
- We find only whole bullets in the detector $\longrightarrow$ bullets always arrive in identical **lumps**
- Result of the experiment plotted in Fig. 1(c)
- $P_{12} \longrightarrow$ probability that a bullet hits the point x

- Repeat the experiment with hole 2 closed $\longrightarrow$ probability distribution P_1 Fig. 1(b)
- Repeat the experiment with hole 1 closed $\longrightarrow$ probability distribution P_2 Fig. 1(b)
- $\boldsymbol{P_{12} = P_1 + P_2 \longrightarrow}$ **no interference**
- A similar experiment with waves

An interference experiment with water waves

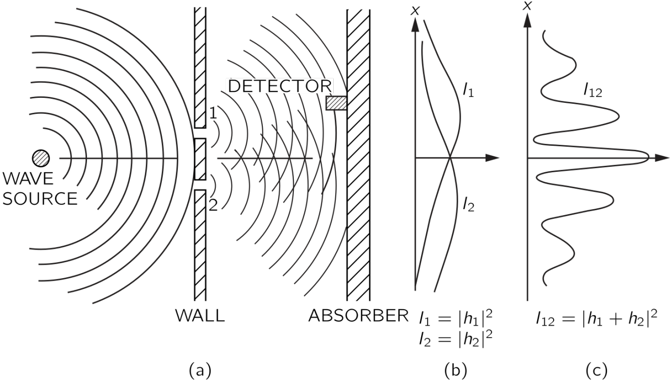


Figure: Interference experiment with water waves

- Circular waves in a shallow trough of water
- Detector $\longrightarrow$ "intensity" of waves
- Intensity $\longrightarrow$ (height of waves)$^2 \equiv$ (amplitude)^2
- Intensity can have "any size" $\longrightarrow$ depends on the motion of the source
- Measure and plot the intensity I_{12} for various values of $x \longrightarrow$ Fig. 2(c)
- Repeat exp. (block 2) $\longrightarrow I_1$, repeat exp. (block 1) $\longrightarrow I_2$ Fig. 2(b)
- $\boldsymbol{I_{12} \neq I_1 + I_2 \longrightarrow}$ **interference**

Instantaneous ht. of wave from hole 1 at detector $\longrightarrow \mathrm{Re}(|h_1|e^{i(\omega t+\phi_1)})$

Instantaneous ht. of wave from hole 2 at detector $\longrightarrow \mathrm{Re}(|h_2|e^{i(\omega t+\phi_2)})$

$h_1, h_2 \longrightarrow$ complex, in general, $I_1 = |h_1|^2, I_2 = |h_2|^2$

Add the waves from 1 and 2,

$$
\begin{aligned}
R &= |h_1|e^{i(\omega t+\phi_1)} + |h_2|e^{i(\omega t+\phi_2)} \\
&= (|h_1|e^{i\phi_1} + |h_2|e^{i\phi_2})e^{i\omega t}
\end{aligned}
$$

$$
\begin{aligned}
I_{12} &= ||h_1|e^{i\phi_1} + |h_2|e^{i\phi_2}|^2 \\
&= (|h_1|e^{i\phi_1} + |h_2|e^{i\phi_2})(|h_1|e^{-i\phi_1} + |h_2|e^{-i\phi_2}) \\
&= |h_1|^2 + |h_2|^2 + |h_1||h_2|(e^{i(\phi_1-\phi_2)} + e^{i(\phi_2-\phi_1)}) \\
&= |h_1|^2 + |h_2|^2 + 2|h_1||h_2|\cos(\phi_1-\phi_2) \\
&= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1-\phi_2)
\end{aligned}
$$

$$\boldsymbol{I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1-\phi_2)}$$

An interference experiment with electrons

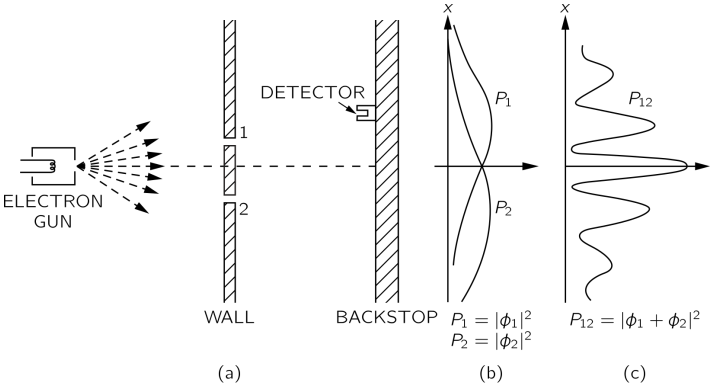


Figure: Interference experiment with electrons

- Detector connected to a loudspeaker
- All clicks from the detector are the same. No half clicks.
- If detector is moved the rate of clicks changes, but the loudness remains the same
- Put two separate detectors at the backstop, one or the other clicks, never both at once
- Electrons always arrive in identical **lumps**
- What is the relative probability that an electron 'lump' will arrive at the backstop at various distances x from the center?
- Probability P_{12} plotted in Fig. 3(c)

- **Proposition A: Each electron either goes through hole 1 or it goes through hole 2.**
- Repeat the experiment with hole 2 closed $\longrightarrow$ probability distribution P_1 Fig. 3(b)
- Repeat the experiment with hole 1 closed $\longrightarrow$ probability distribution P_2 Fig. 3(b)
- $\boldsymbol{P_{12} \neq P_1 + P_2 \longrightarrow}$ **There is interference, proposition A is false**
- In analogy with expt. on waves we may write, $\boldsymbol{P_1 = |\phi_1|^2, P_2 = |\phi_2|^2, P_{12} = |\phi_1 + \phi_2|^2}$
- Electrons sometimes behave like particles "lumps" and sometimes like waves "interference"

Follow the electrons

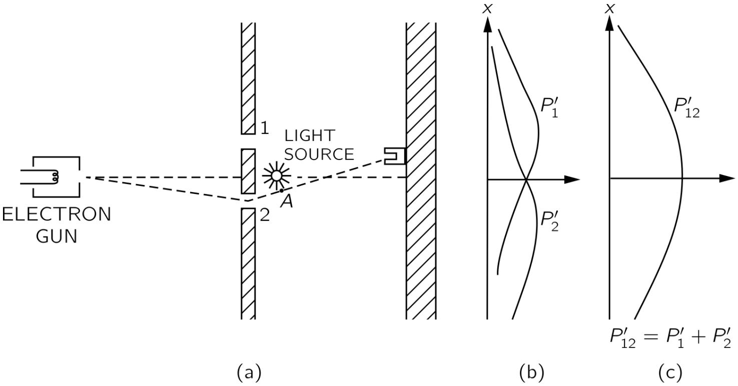


Figure: Another interference experiment with electrons

- Is Proposition A true?
- Light source placed behind wall, between the two holes
- Electric charges scatter light
- One click at the detector $\longrightarrow$ light flash either near 1 **or** 2
- Electrons go either through one hole or the other
- Proposition A is true

- Now, count the electrons that arrive at the detector **and** keep track of which hole they went through
- For the electrons from 1 we get P_1' Fig. 4(b)
- For the electrons from 2 we get P_2' Fig. 4(b)
- Probability P_{12}' that an electron will arrive at the detector passing through **either** hole $\longrightarrow$ $\boldsymbol{P_{12}' = P_1' + P_2'}$
- Watch the electrons and determine their paths $\longrightarrow$ no interference
- Light disturbs the motion of the electrons $\longrightarrow$ erases interference pattern

- What if we **decrease** the brightness of the source?
- Flashes of light scattered by electrons **does not** get weaker, but sometimes there is a click and no flash
- Electron has passed a hole without being seen $\longrightarrow$ light also acts like electrons $\longrightarrow$ knew it was wavy, find that it is also lumpy $\longrightarrow$ lumps called **photons**
- Turn down intensity $\longrightarrow$ decrease the rate of emission and not the size of photons
- When there is a click at the detector
  1. electron has passed through 1
  2. electron has passed through 2
  3. electron not seen

- For 1 $\longrightarrow P'_1$, for 2 $\longrightarrow P'_2$, for 1 or 2 $\longrightarrow P'_{12}$, for 3 $\longrightarrow P_{12}$
- Can we see the electrons without disturbing them?
- $p = h/\lambda \longrightarrow$ greater the wavelength, lesser the momentum $\longrightarrow$ use infrared light or radiowaves
- Limitation of resolving two separate spots $\longrightarrow$ distance between spots $\approx$ wavelength of light
- $\lambda \approx$ distance between holes $\longrightarrow$ no longer able to tell whether electron has passed through 1 or 2 $\longrightarrow P'_{12}$ begins to look like P_{12}
- It is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electrons enough to destroy the interference pattern $\longrightarrow$ **Heisenberg uncertainty principle**

- What about proposition A? True? False?
- Depends on the observables measured by the observer

An important difference between classical mechanics and quantum mechanics

- Classical mechanics $\longrightarrow$ deterministic
- Quantum mechanics $\longrightarrow$ probabilistic

Quantum mechanics $\longrightarrow$ foundation behind new age technologies like quantum logic clocks, quantum cryptography, quantum entanglement enhanced microscopes, quantum computation etc.

Thank you!